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Reference: NASA Research Grant NGR 41-002-001, University of South Carolina, Columbia, S. C.

From: Dr. T. F. Bridgland, Jr., Principal Investigator

Subject: Semi-annual progress report.

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Introduction

The content of this report is divided into two parts. The first is a technical report of progress on the research conducted under this grant during the period January 15, 1965 - July 15, 1965 and the second is a financial report of grant fund expenditures made in connection with this grant during the period January 15, 1965 - June 30, 1965.

PART I - REPORT ON RESEARCH PROGRESS

During the six months period ending July 15, 1965, the major portion of the research effort under this grant has been devoted to a consideration of items (1) and (2) of [1]*.

Substantial progress has been made in the study of the problem proposed in item (2) of [1]. In particular it has been established that, under wide conditions, the set B of that item has a non-void interior,

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* Numbers in square brackets refer to references listed at the end of Part I.

thus causing B to have positive Lebesgue measure. In addition, some results have been obtained concerning the continuity of the final time functional, $t^*(t_o, x_o, u)$.

In connection with item (1) of [1], it became clear at an early stage of our research effort that a useful resolution of the optimal feedback control problem proposed in that item must involve fundamental assumptions concerning the structure of the control set. Development of the Carathéodory approach to feedback problems in the absence of such assumptions was pushed as far as is possible by the principal investigator in [2]. In that reference, the Carathéodory methods developed in [3] were used to establish sufficient conditions for the existence of a feedback control law in general control problems of Lagrange type. Any further extension of the Carathéodory methods necessarily involves the determination of sets of controls whose structure guarantees the satisfaction of these sufficient conditions.

In view of this, control sets involving certain assumptions on their compactness and convexity have been introduced into the problem. These assumptions, together with a particularly useful reformulation of the results of [2], has permitted the application to the feedback control problem of a fairly large body of fundamental results from the theory of convex sets. Whereas we have obtained a number of interesting - albeit incomplete - results by means of this approach, the convex set set theory to which we have referred above is by no means so well developed per se as to permit immediately a complete solution to the control problem under investigation. The parts of convex set theory which are the most interesting from the point of view of feedback control theory still possess several gaps and the problems associated with attempting to fill in those gaps have consumed a substantial proportion of the time devoted to research during this reporting period.

In conclusion, the progress made in research under this grant during the last six months has been most encouraging to the principal investigator.

References

1. Problems Arising in the Theory of Optimal Feedback Control, Sept. 1963, proposal submitted to NASA substantiating funding under grant NGR 41-002-001, p.16, IV, "Statement of Work";

2. T. F. Bridgland, Jr., On the existence of optimal feedback controls. II., SIAM Journal, Series A on Control, vol. 2, 1965, pp. 137 - 150;

3. T. F. Bridgland, Jr., On the existence of optimal feedback controls, ibid., vol. 1, 1963, pp. 261-274.